Quantum Mechanics and 3*N*-Dimensional Space

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I maintain that quantum mechanics is fundamentally about a system of *N* particles evolving in three-dimensional space, not the wave function evolving in 3*N*-dimensional space.

1. Introduction. What is quantum mechanics fundamentally about? I believe that quantum mechanics is fundamentally about particles: quantum mechanics describes the behavior of systems of particles evolving in three-dimensional space (or, if you prefer, four-dimensional space-time). (At least, that's what I would say for nonrelativistic quantum mechanics; for the relativistic case, I'd say that quantum mechanics is fundamentally about fields, existing in four-dimensional space-time. The ontological issue I'm interested in is fundamentally the same in the relativistic and nonrelativistic cases, so I'll stick with the nonrelativistic case for simplicity.) Some physicists and philosophers disagree with my claim that quantum mechanics is fundamentally about particles: they would hold that quantum mechanics is fundamentally about *the wave function*, construed as a really existing field evolving in a 3*N*-dimensional space (where *N* is the number of particles standardly thought to exist in three-dimensional space).

I don't believe in the wave function, and the point of this paper is to convince you that you shouldn't believe in it either. Now, of course I think that the wave function is a useful mathematical tool; it is useful to describe systems as having quantum states, represented by wave functions. But as a matter of *ontology*, the wave function doesn't exist; or, at least, the wave function is no more real than the numbers (such as 2 or π) that

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go into the equations used to describe quantum systems. The wave function is, at best, an abstract entity, and if you're a nominalist about abstract entities as I am, then you should be happy to say that the wave function doesn't exist.

I'll start by spelling out in a bit more detail my preferred picture of what quantum mechanics is about, and then I'll suggest why people might not be happy with it. This will lead us to contemplate various alternatives, and for each of the alternatives I'll point out some problems. After you have seen the problems that the alternatives face, I hope you will join me in being sanguine about my preferred picture.

2. The Wave Function Is Represented by a Property. On my picture of what quantum mechanics is about, quantum mechanics is about particles, and systems of particles, all evolving in three-dimensional space. Quantum mechanics tells us a bit about the properties of these particles, but to really get a full story, one needs an interpretation of quantum mechanics. A modal interpretation, for example, will specify the circumstances under which these particles have definite properties. The many-worlds interpretation would specify that there are actually different worlds, and the particles in the different worlds have different properties. For whatever interpretation you pick, the interpretation gives a fuller account of what properties these particles have.

Where does the wave function fit into all this? The wave function is a representation of the quantum state of a closed system. As long as this quantum state is pure—and, since I'm restricting the discussion to closed systems, it's reasonable to assume that it is pure—then it's the eigenstate of some observable. I endorse the eigenstate-to-eigenvalue half-link—if not for every quantum system, then at least for the system of all the particles in the universe (or if the universe is too big for you, then whatever closed system you are interested in). The eigenstate-to-eigenvalue half-link holds that if the system is in an eigenstate of some observable, then the system actually has the property represented by the eigenvalue associated with that eigenstate. So, since the quantum state of all the particles in the universe (or whatever closed system you're interested in) is an eigenstate of some observable, then by the eigenstate-to-eigenvalue half-link, the system actually has the property represented by the eigenvalue. That's how the wave function fits into my picture: the wave function doesn't exist on its own, but it corresponds to a property possessed by the system of all the particles in the universe (or whatever closed system you're interested in).

I find this picture perfectly natural and unproblematic; were it not for the fact that smart people have endorsed pictures incompatible with mine, I would have nothing more to say. In fact, it's not even obvious to me why people are unhappy with my picture. Perhaps they don't want to endorse the eigenstate-to-eigenvalue half-link, even in the restricted way in which I've endorsed it. Perhaps they are unhappy with holistic properties; perhaps they want all properties of systems to be based on properties of the component particles. Perhaps they are unhappy with the wave function being represented by a corresponding property; perhaps they want the wave function itself to physically exist. Perhaps they are unhappy with the fundamental dynamical law of quantum mechanics being based on a property of the evolving system.

To be honest, I'm not moved by any of these reasons, but perhaps you are, or perhaps you have another reason in mind that I haven't thought of, or perhaps you're just wondering what the other options are. So let's look at what else is out there.

3. Other Viable Ontologies. I'll start by considering two fallback positions—positions that I would endorse if for some reason I had to give up the position I've sketched above. Then I'll take up the hard-core position that holds that the fundamental ontology of the world is the wave function (and perhaps other objects) evolving in $3N$-dimensional space.

My first fallback position is to reject not only the existence of the wave function but also the existence of the property possessed by the system of all the particles in the universe that corresponds to the wave function. The wave function would just be a part of the dynamical equation used to describe how systems evolve.

This would lead to a somewhat different conception of how laws of nature in quantum mechanics work. Schrödinger's (1926) equation has a parameter ψ, representing the wave function. According to the position I endorsed in the previous section, the value of that parameter is determined by a property of the system of particles. According to the proponents of the wave function ontology, the value of that parameter is determined by the wave function itself, evolving in a $3N$-dimensional space. But according to the fallback position I'm presenting now, there's nothing in the universe that determines the value of that parameter; instead the value of that parameter has to be a part of the equation itself.

This would mean that systems with different wave functions have different dynamical laws describing their evolution; quantum mechanics would have an infinite number of incompatible dynamical laws. For one who takes laws of nature seriously, this might be problematic. I, however, do not take laws of nature seriously; I'm sympathetic to the position of van Fraassen (1989). I endorse the semantic view of scientific theories, where a theory is described by a family of mathematical models (together with theoretical hypotheses that specify how these models relate to the world). On the semantic view, the different dynamical laws would simply correspond to different sets of models. It doesn't matter that the laws vary across models; the laws are simply heuristically useful in describing the various sets of models.

I know that some people will be unhappy with this; some people will hold that the wave function *has* to correspond to something physical in the world. This brings me to my second fallback position, what I will call the *mixed* ontology. A proponent of this ontology holds that there is both a three-dimensional space in which the N particles evolve and also a $3N$-dimensional space in which the wave function evolves.

I have nothing definitive to say about this ontology, other than that it strikes me as strange. We have two disconnected spaces, with presumably no causal connection between the particles in the one space and the field in the other space, and yet the stuff in the two spaces is evolving in tandem. Presumably there is a nomic connection between the stuff in the two spaces, which supports counterfactuals of the following form: if the stuff in one space had evolved differently, the stuff in the other space would have evolved differently. But having that nomic connection without a causal connection makes it all the more mysterious how these spaces are associated with each other.

There is a variant of the mixed ontology worth considering: one could hold that, instead of a separate three-dimensional space and a $3N$-dimensional space, in fact both of these spaces are hypersurfaces in a $(3N + 3)$–dimensional space. This would be a first step toward establishing something like a causal connection: at least now the particles and the wave function field are evolving in the same space. But still, it is mysterious what the mechanism is for the connection between the two hypersurfaces.

A proponent of the mixed ontology might say that my criticism is unfair: in my preferred picture, there is no causal connection between the system of particles and the property possessed by the system of particles that corresponds to the wave function. My reply is that there does not need to be a causal connection: it is commonplace that the evolution of a system of particles depends on what properties that system has.

4. The Wave Function Ontology. Let's now turn to the ontology that holds that quantum mechanics is fundamentally about the wave function, evolving in $3N$-dimensional space. Interestingly, this picture was explicitly rejected when quantum mechanics was first being developed. I will discuss the reactions of Schrödinger, Lorentz, and Bohm.

Schrödinger started out trying to interpret the wave function realistically. For example, in an early paper on wave mechanics, he writes:

> The true mechanical process is realised or represented in a fitting way by the *wave processes* in *q*-space, and not by the motion of *image points* in this space. (1926, 25)

Here *q*-space is configuration space. Schrödinger's claim is made in the context of a discussion of one-particle systems, where configuration space is just three-dimensional space. What about a multiparticle system, though? Schrödinger considers a two-particle system late in the paper but has only one sentence about the physical representation of the six-dimensional wave function:

> The direct interpretation of this wave function of *six* variables in *three*-dimensional space meets, at any rate initially, with difficulties of an abstract nature. (39)

Schrödinger wants to interpret the mechanical processes realized or represented by the wave function as taking place in three-dimensional space, but he does not see how this can be done.

Lorentz picks up on this problem with multiparticle systems. In 1926, Lorentz wrote a letter to Schrödinger, in which he says:

> If I had to choose now between your wave mechanics and the matrix mechanics, I would give the preference to the former, because of its greater intuitive clarity, so long as one only has to deal with the three coordinates *x, y, z*. If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics. (Lorentz, in Przibram 1967, 44)

Schrödinger kept trying to develop an ontology for the wave function, but by 1935 he had conceded the point:

> I am long past the stage where I thought that one can consider the ψ-function as somehow a direct description of reality. (Schrödinger, in Fine 1996, 82)

This unhappiness with the wave function ontology was expressed not just when quantum mechanics was first being formulated, but also in later interpretative work. For example, David Bohm writes of his point particle–based theory:

> While our theory can be extended formally in a logically consistent way by introducing the concept of a wave in a 3*N*-dimensional space, it is evident that this procedure is not really acceptable in a physical theory. (1957, 117)

While Bohm does not say it explicitly, one gathers that the reason it is not acceptable to interpret his theory in that way is that such an understanding does not match the world as we experience it. While it is mathematically viable to represent the theory as consisting of objects evolving in 3*N*-dimensional space, it is not *physically* viable, because 3*N*-dimensional space is not an accurate representation of the physical, three-dimensional world.

5. Albert's Defense. I will now take up the arguments of two contemporary defenders of the wave function ontology, David Albert and Peter Lewis. I'll discuss Albert in this section and Lewis in the next.

Albert is explicit in his endorsement of the wave function ontology and draws the following consequence:

> the space *we* live in, the space in which any realistic understanding of quantum mechanics is necessarily going to depict the history of the world as *playing itself out* . . . is *configuration*-space. And whatever impression we have to the contrary (whatever impression we have, say, of living in a three-dimensional space, or in a four-dimensional space-time) is somehow flatly illusory. (1996, 277)

We think that we live in three-dimensional space, but we are mistaken; in fact we live in 3*N*-dimensional space. If this claim were true, this would be the most radical revision of people's everyday understanding of the world ever engendered by science, far surpassing any other scientific revolution in our worldview. It is this radically revisionary nature of the wave function ontology hypothesis that leads me to assign a low prior probability to the hypothesis; the evidence would have to be very strong to lead me to accept the hypothesis.

In fact, I think that the radically revisionary nature of the wave function ontology can be used to generate an argument against the ontology. My argument relies on a pragmatic maxim, but it's a maxim that has had much force in the history of science: one should not accept theories that radically revise people's everyday understanding of the world when there are other, at least equally acceptable, theories that do not entail such extreme revision. (The other theories should be either equally or more acceptable on grounds such as simplicity, lack of ad hoc–ness, and compatibility with other parts of science; they would of course be more acceptable on the ground of compatibility with everyday understanding.)

In conversation, I have found that some people disagree with my claim that the wave function ontology, as presented by Albert, is radically revisionary with respect to our everyday understanding of the world. In my opinion, the point is obvious, but let me try to spell it out in more detail. The reason the wave function ontology entails a radical revision of our everyday understanding of the world is that our everyday understanding holds that the world consists of objects with length, breadth, and depth evolving in a three-dimensional space. According to the wave function ontology, claims that objects exist in three-dimensional space are, strictly speaking, false: at the level of fundamental reality, there is no three-dimensional space; there is only $3N$-dimensional space. Our everyday understanding of the world is not simply that the world appears to us to have objects evolving in three-dimensional space; our everyday understanding is that the world *does* have objects evolving in three-dimensional space. The wave function ontology may be able to account for the appearances, but it is radically revisionary with respect to how we take things to actually be.

In this respect the wave function ontology is similar to the brain in the vat scenario: in the brain in the vat scenario, we think that the world around us is a certain way, but it turns out that we are radically mistaken about the basic facts regarding the world around us; we are actually all brains in vats. In fact, in some ways the wave function ontology is even more radical than the brain in the vat scenario: in the brain in the vat scenario, at least we are correct in thinking that we have brains existing in three-dimensional space. According to the wave function ontology, even that is incorrect; all that really exists is stuff evolving in $3N$-dimensional space. Just as we think that there is strong prima facie reason to reject the brain in the vat scenario, because of its radically revisionary implications for commonsense ontology, so there is a strong prima facie reason to reject the wave function ontology.

The "prima facie" qualifier is important. The pragmatic maxim I endorsed above requires that there be other, otherwise equally acceptable, theories that do not entail such extreme revision. This is where the position I endorsed in Section 2 comes into play: having the wave function correspond to a property possessed by the system of particles is a perfectly reasonable ontology and does not require any radical revision of people's commonsense understanding of the world.

The argument I have given above is, in my opinion, the strongest argument one can give against Albert's version of the wave function ontology. Before moving on to Lewis, however, I want to look in more detail at Albert's version of the ontology and raise one more problem for it.

Given that space is really $3N$-dimensional, according to Albert, how could there possibly be beings in such a world that think they're living in a three-dimensional world? Albert argues that the *Hamiltonian* of the equation of motion for the wave function determines in what way the objects in $3N$-dimensional space represent a (hypothetical) world with multiple particles. He writes:

> a quantum-mechanical world with [one sort of Hamiltonian] will

> appear to its not-too-closely-looking inhabitants to have *two* dimensions, and . . . a quantum-mechanical world with [another sort of Hamiltonian] will appear to *its* not-too-closely-looking inhabitants (just as *our own* does) to have *three*. (1996, 282)

Albert's argument for this is as follows. Suppose that the wave function ontology is true, and suppose that the Hamiltonian is one that would describe a normal sort of interaction between N particles in three-dimensional space. Albert writes that, given such a Hamiltonian,

> *one particular* hypothesis . . . can be distinguished as uniquely natural and reasonable and elegant and whatever else you might like: the hypothesis that we are looking at a *three*-dimensional space, in which [N] distinct material particles are floating around; the hypothesis (that is) that the potential terms in this Hamiltonian represent an interparticle force whose intensity depends on the *distance* between the particles in question. (280–281)

Albert's conclusion is that the quantum Hamiltonian determines that the world appears three-dimensional to its inhabitants, even though such appearances are nonveridical.

My problem with this argument is that it's simply not clear to me that mental states would exist in the world as described by Albert. I grant that the Hamiltonian of the wave function naturally corresponds to particles interacting in a normal way in three-dimensional space. But the Hamiltonian also corresponds to particles interacting in an abnormal way in a space. One possibility is the one Albert points out: the Hamiltonian corresponds to particles interacting in an abnormal way in two-dimensional space (assuming $3N$ is even). Another possibility is that the Hamiltonian corresponds to particles interacting in an abnormal way in 17-dimensional space (assuming $3N$ is a multiple of 17). Yet another possibility is that the Hamiltonian corresponds to particles interacting in an abnormal way in standard three-dimensional space: this would depend on which dimensions in $3N$-dimensional space are chosen to correspond to which particles in three-dimensional space.

There is nothing to picking out one of these correspondences as the right one; all the correspondences are equally hypothetical. If the correspondence Albert likes were the right one—if there really were a three-dimensional space corresponding to the $3N$-dimensional space, with particles evolving in the normal way—then it is clear that mental states would exist in that world. But if one of the other correspondences I discussed were the right one—for example, if there really were a two-dimensional space corresponding to the $3N$-dimensional space, with particles evolving in an abnormal way—then it is clear that mental states would *not* exist in that world. Presumably Albert would say that the mental states that would exist on each possible correspondence actually do exist. But it seems equally reasonable to say that no mental states would exist, since all the correspondences are just hypothetical.

I don't know of any way to determine whether or not mental states would exist in the world as described by Albert, and Albert doesn't give an argument for his preferred view; he just takes for granted that there are inhabitants of the 3*N*-dimensional world. I conclude that this second problem I have raised for Albert's version of the wave function ontology is not a decisive one, but it does show that more work would need to be done to provide an adequate defense of the ontology. (For a somewhat different discussion of this problem, see Monton [2002].)

6. Lewis's Defense. Peter Lewis also wants to defend the wave function ontology, but he is aware of the two problems I've raised for Albert's view. Lewis attempts to defend a version of the wave function ontology that gets around both of these problems.

Here is the fundamental claim that Lewis endorses:

> There is a sense in which the wavefunction is a three-dimensional object living in a three-dimensional space, and a sense in which it is a 3*N*-dimensional object living in a 3*N*-dimensional space. (2004, 726–727)

If this claim were true, then the wave function ontology wouldn't be radically revisionary; we can stick with our commonsense understanding of the world as three-dimensional, even given the wave function ontology. Moreover, if this claim were true, there wouldn't be a question of whether mental states exist; one three-dimensional space would be uniquely selected, and the mental states associated with that three-dimensional space would unproblematically exist. But unfortunately for the wave function ontology, I think that Lewis's claim is not true; I will argue that the wave function has to be understood as existing in 3*N*-dimensional space.

Lewis's argument for his claim is based on his thesis that there are two ways of understanding the dimensionality of a space. He writes:

> So what does it mean to claim that a system has a certain number of dimensions? The most straightforward answer is that it requires that many independent coordinates to parameterize the properties of the system. (2004, 726)

This answer yields the conclusion that the wave function is a 3*N*-dimensional object since "the state of a quantum system at a time is specified by the distribution of wavefunction properties over the possible values of 3*N* coordinates."

Lewis considers an alternative answer, though, "couched in terms of coordinate transformations rather than the number of parameters." As we will see, this alternative answer leads to the result that the wave function is a three-dimensional object.

Lewis starts his defense of the alternative answer by arguing for the following:

> In order for the form of the quantum mechanical Hamiltonian to be invariant under the choice of coordinate system, specifying the origin and direction for three axes must suffice to specify the coordinate system for the configuration space. (2004, 726)

Lewis's argument for this is as follows. If the Hamiltonian is invariant under arbitrary origin-shifting coordinate transformations, then for any location in 3*N*-dimensional space one could pick that location to be the origin. But this would represent an *N*-particle system in which all particles are at the same location in three-dimensional space. It follows that, for the Hamiltonian to be invariant, the only possible coordinate transformations are those that perform the same transformation on each triple in Albert's preferred grouping of coordinates into ordered triples.

Lewis's argument here is complicated, but up to this point I'm inclined to believe that it's correct. Lewis suggests, though, that the restriction on allowed coordinate transformations means that "configuration space in fact has three dimensions rather than 3*N*" (2004, 725). Here I think Lewis is going too far.

I maintain that there is only one correct way of understanding the dimensionality of a space, and on that understanding the wave function definitively exists in 3*N*-dimensional space. I do not, however, want to endorse Lewis's "independent coordinate" rationale for why the wave function exists in 3*N*-dimensional space. I'm not sure what work the "independent" qualifier is doing, but my worry is that it doesn't help to get around the fact that a location in a space of any (finite) dimensionality can be picked out with a single coordinate. (Regardless of the dimensionality of the space, there are only continuum many points, and hence the set of points can be put in one-to-one correspondence with the set of real numbers.) Thus, the state of a quantum system can be specified by the distribution of properties over the values of a single coordinate; it doesn't take 3*N* coordinates.

The reason I maintain that there is only one correct way of understanding the dimensionality of a space is that mathematicians have an agreed-upon account of the dimensionality of a space, and I don't know of any good arguments against their account. Here is one version of the standard mathematical account:

a. the empty set has dimension -1,
b. the dimension of a space is the least integer n for which every point has arbitrarily small neighborhoods whose boundaries have dimension less than n. (Hurewicz and Wallman 1941, 4)

This account yields the desired result that, if the wave function exists, it exists in $3N$-dimensional space.

On this account, the dimensionality of a space is a topological matter. It follows that how many dimensions a space has is independent of the laws of nature that describe what goes on in that space. Lewis's coordinate transformation–based account does not respect this standard topological understanding of the concept of dimensionality. What coordinate transformations are allowed on a space is a scientific matter, not a topological one. In fact the very idea that the Hamiltonian should be invariant under the choice of coordinate system is a scientific requirement, not a topological one. The mathematician's concept of dimensionality simply doesn't depend on facts about coordinate transformations. Perhaps Lewis would maintain that the mathematician's account of dimension is not the one that is relevant to understanding the dimension of a physical space, but in the absence of any such argument I maintain that we should rely on the standard mathematical understanding of dimensionality.

I conclude that there is no legitimate way to understand the wave function as a three-dimensional object; the wave function is unequivocally $3N$-dimensional. It follows that the first problem I raised for Albert still stands: the wave function ontology is radically revisionary with respect to our commonsense understanding.

What about the second problem I raised for Albert? Will mental states exist in the world in which Lewis's version of the wave function ontology holds? Here Lewis's version is more promising than Albert's: Lewis attempts to build into the $3N$-dimensional space enough structure such that there is a unique correspondence between the wave function and a (hypothetical) system of N particles evolving in three-dimensional space. I will grant Lewis that he succeeds in establishing this unique correspondence. But does it follow that mental states clearly exist on Lewis's version of the wave function ontology?

No, it does not. If the correspondence were actually to be instantiated—if there really were a system of N particles evolving in three-dimensional space corresponding to the $3N$-dimensional wave function—then it is clear that mental states would exist in that world. But given that the correspondence is not instantiated, we have a version of the problem raised above for Albert: it's not clear whether mental states would exist.

To make this point salient, consider the classical case. First consider a three-dimensional space with N point particles, and suppose that the particles evolve in such a way that mental states exist in this world. Now consider a 3*N*-dimensional space with a single point particle, where the point particle evolves in such a way that it represents the evolution of the *N* particles in three-dimensional space. Now, if that 3*N*-dimensional space with that single point particle is all that exists, would mental states exist in that world? Could billions of different mental states all supervene on a single point particle? It's simply not obvious that they could; it's simply not clear that mental states would exist in that world.

To sum up: on both Albert's and Lewis's versions of the wave function ontology, the ontology is radically revisionary with respect to our commonsense understanding of the world, and it's simply not clear whether mental states would exist given their ontology. This leads to the natural question: Is there a better ontology? I maintain that the ontology I presented in Section 2 is clearly better: the wave function can correspond to a property possessed by the system of all the particles in the universe (or whatever closed system you're interested in).

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